MATHEMATICS

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Unit Outline

12.1   Frequency Distribution
12.2   Measures of Central Tendency

Students Learning Outcomes

After studying this unit, the students will be able to:
- Define frequency, frequency distribution.
- Construct frequency table.
- Construct a histogram representing frequency table.
- Describe measures of central tendency.
- Calculate mean (average), weighted mean, median and mode for ungrouped data.
- Solve real life problems involving mean (average), weighted mean, median and mode.

12.1   FREQUENCY DISTRIBUTION

12.1.1 Definition of frequency and frequency distribution

Frequency

     The number of times a particular observation occurs in a given set of data is called its frequency. It is denoted by f.

Example 12.1   The following are the marks of 20 students in a class test out of 25 marks: 16, 18, 15, 18, 13, 15, 18, 16, 19, 18, 16, 15, 20, 16, 16, 18, 19, 18, 16, 18. Show the frequency of each observation.

Solution

     Frequency of the marks is shown below in the table.

MARKS	FREQUENCY (*f*)
13	1
15	3
16	6
18	7
19	2
20	1
Total	**20**

In the above table, we observe that

Frequency of 13 is 1

Frequency of 15 is 3

Frequency of 16 is 6

Frequency of 18 is 7

Frequency of 19 is 2

Frequency of 20 is 1

Frequency Distribution

A tabular arrangement of given numerical data showing the frequency of the different observations or classes is called frequency distribution.

Method by definition (Director method)

In direct method, we enlist the observations against respective classes, number the observations will be the frequency of that class. The other process of forming a frequency table is the same as given tally bar method. The method is illustrated by the following example.

Example 12.2 Construct the frequency distribution of the marks obtained by 25 students in English test out of 50 marks by

(i)   direct method   (ii)   tally method

15, 20, 44, 18, 23, 25, 29, 18, 25, 34, 29, 20, 25, 34, 37, 20, 23, 25, 29, 34, 23, 25, 37, 29, 25

Solution

(i) Direct method

Marks	Observations	Frequency (f)
15	15	1
18	18, 18	2
20	20, 20, 20	3
23	23, 23, 23	3
25	25, 25, 25, 25, 25, 25	6
29	29, 29, 29, 29	4
34	34, 34, 34	3
37	37, 37, 27	2
44	44	1
Total		**25**

(ii) Tally method

Marks	Tally	Frequency (f)
15			1
18				2
20					3
23					3
25	‖‖|	6
29						4
34					3
37				2
44			1
Total		**25**

12.1.2 Construction of Frequency Table

For construction of frequency distribution table, following steps should be kept in mind.

(i)   Choose the smallest and the largest value in a set of given data.

(ii) Find the difference of the largest and the smallest value which is called range.

(iii) If number of classes are not given, then decide about the number of classes.

(iv) Divide the range by number of classes.

In this way we find the size of class interval.

$$\text{Size of class interval} = \frac{\text{largest value} - \text{smallest value}}{\text{number of Classes}}$$

In order to make frequency distribution table, we put a tally marks | opposite to the observation Tallies are marked in bunches of 5.

If an observation repeats 2 times, we put two tallies ||. In other words we put tally marks as many as the observation is repeated. For an observation repeated five times, we put 4 tallies and fifth tally is diagonally cross the four tallies ||||. If an observation is repeated 11 times, we can show as |||| |||| |.

The method is illustrated by the following example.

Example 12.3 The weights in kg of 40 students selected from a school are given below construct the frequency table.

40, 35, 42, 35, 45, 37, 41, 33, 37, 30, 28, 29, 30, 32, 33, 35, 31, 30, 36, 28, 39, 37, 45, 36, 38, 35, 28, 31, 34, 39, 36, 35, 41, 28, 34, 31, 30, 28, 29, 37

Solution

Smallest Value = 28

Largest Value = 45

Let the number of classes = 6

$$\text{Size of class interval} = \frac{\text{largest value} - \text{smallest value}}{\text{number of classes}}$$

$$\text{Size of class interval} = \frac{45-28}{6} = \frac{17}{6} = 2.83 = 3 \text{ (approximately)}$$

Frequency Table

Class Interval	Tally	Frequency (f)
28 – 30	Ⅲ Ⅲ Ⅰ	11
31 – 33	Ⅲ Ⅰ	6
34 – 36	Ⅲ Ⅲ	10
37 – 39	Ⅲ Ⅰ	7
40 – 42	Ⅰ Ⅰ	4
43 – 45	Ⅰ	2
Total		**40**

Class limits

The smallest and largest values in any given class are called its class limits. For example, the class 56 – 60 starts with 56and ends at 60. The starting point 56 is called lower class limit and the end point 60 is called upper class limit.

Class Boundaries

For making class boundaries, take the difference of the lower class limit of second class and upper class limit of the first class then divide by 2. The obtained number is subtracted from each lower class limit and added in each upper class limit of the data.

Example 12.4 Make class boundaries of the data given in example 12.3 and reconstruct the table.

Solution

Lower class limit of second class = 31

Upper class limit of first class = 30

Difference = 31 – 30 = 1

Now, $\dfrac{1}{2} = 0.5$

We subtract 0.5 from each lower class limit and add 0.5 in each upper class limit.

Classes	Class Boundaries	Frequency (f)
28 – 30	27.5 – 30.5	11
31 – 33	30.5 – 33.5	6
34 – 36	33.5 – 36.5	10
37 – 39	36.5 – 39.5	7
40 – 42	39.5 – 42.5	4
43 – 45	42.5 – 45.5	2
Total		**40**

12.1.3 Histogram

Histogram is a graph of the frequency distribution in which class boundaries are taken along x-axis with a suitable width of class and adjacent bars are erected to show the frequencies along y-axis.

In histogram the width of classes are equal but height of the bars are in proportional to the size of the corresponding frequency.

Example 12.5 The marks in Urdu of 36 students out of 75 are as under:

54, 58, 48, 57, 48, 45, 46, 49, 52, 45, 49, 50, 48, 46, 45, 45, 38, 39, 62, 59, 53, 37, 42, 42, 43, 46, 48, 39, 45, 40, 42, 40, 38, 38, 35, 58

Form a frequency distribution of the data and draw histogram to represent it.

Solution Smallest value = 35

Largest value = 62

Let the number of classes = 7

$$\text{Size of class Interval} = \frac{\text{largest value} - \text{lowest value}}{\text{number of classes}}$$

$$= \frac{62 - 35}{7} = \frac{27}{7} = 3.86 \approx 4$$

Class Interval	Class Boundaries	Tally	Frequency (f)
35 – 38	34.5 – 38.5	||||	4
39 – 42	38.5 – 42.5	⦀⦀ |||	8
43 – 46	42.5 – 46.5	⦀⦀ ||||	9
47 – 50	46.5 – 50.5	⦀⦀ ||	7
51 – 54	50.5 – 54.5	|||	3
55 – 58	54.5 – 58.5	|||	3
59 – 62	58.5 – 62.5	||	2
Total			**36**

34.5 – 38.5 – 42.5 – 46.5 – 50.5 – 54.5 – 58.5 – 62.5
Class Boundaries
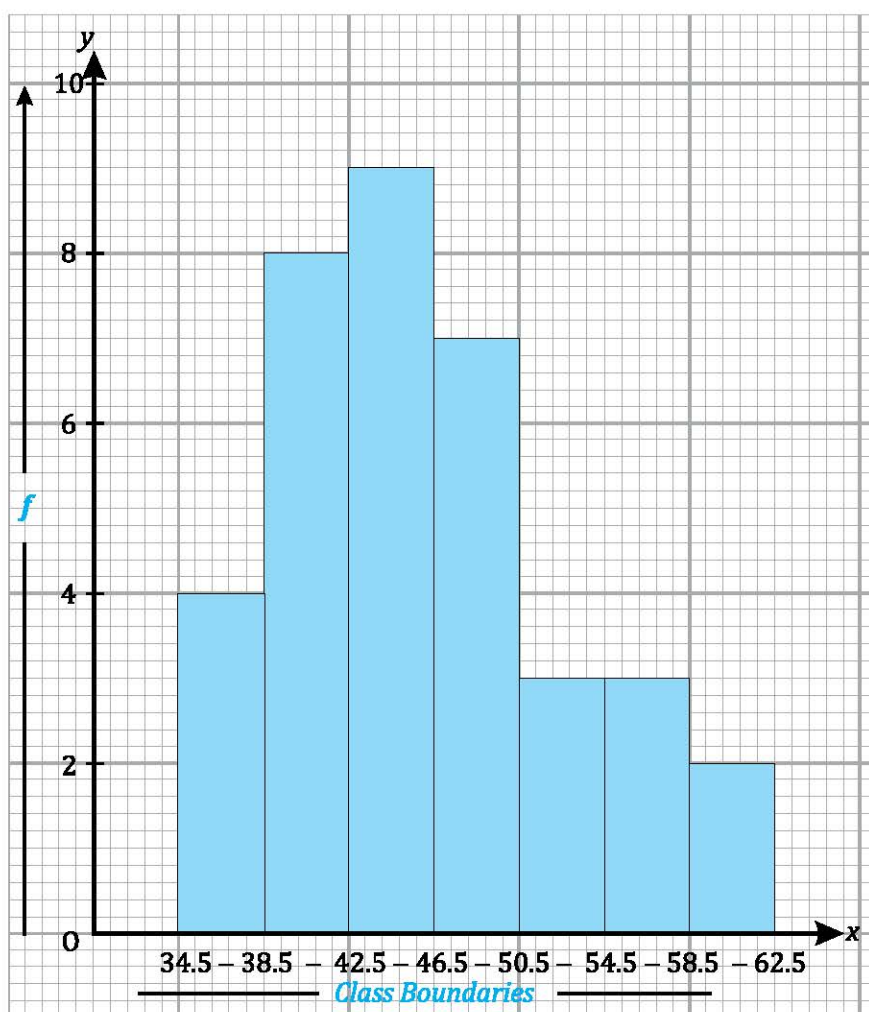
EXERCISE 12.1

1. The following values relate to the number of members in various families. Construct a frequency distribution by taking one as size of class interval.
8, 9, 9, 7, 6, 5, 4, 2, 3, 4, 6, 5, 6, 11, 1, 6, 9, 8, 7, 6, 5, 6, 3, 5, 7, 7, 10, 11, 12, 1, 4, 5, 3, 5, 6

2. The following data give the index numbers of 45 commodities in a certain year. Construct the frequency distribution.
128, 105, 106, 147, 127, 129, 140, 120, 129, 108, 110, 143, 109, 138, 113, 136, 100, 104, 116, 144, 109, 100, 133, 108, 110,
141, 115, 119, 124, 145, 114, 114, 106, 113, 134, 117, 151, 123, 118, 111, 119, 113, 138, 120, 112

3. Tabulate the following marks into a frequency distribution taking 10 as size of class interval.
    82, 62, 85, 105, 116, 91, 83, 99, 52, 76, 84, 89, 77, 104, 101, 107, 62, 46, 109, 74, 103, 95, 90, 118, 52, 88, 101, ,96, 72, 56, 64, 110, 97, 59, 62, 96, 87, 78, 74, 99

4. Form a frequency distribution of the following data by taking the classes as 46 – 50, 51 – 55 etc.
    78, 51, 67, 77, 82, 72, 55, 64, 61, 65, 46, 48, 50, 52, 55, 54, 59, 64, 52, 47, 66, 71, 69, 77, 82, 85, 87, 89, 56, 63, 74, 79, 58, 49, 67, 84, 64, 78, 62, 66, 75, 51, 58, 61

5. Draw histogram for the following frequency distribution.

Class Intervals	Frequency
16 – 25	5
26 – 35	7
36 – 45	19
46 – 55	24
56 – 65	17
66 – 75	14
76 – 85	4

6. Draw histogram for the following frequency distribution.

Class Intervals	Frequency
11 – 20	3
21 – 30	8
31 – 40	18
41 – 50	25
51 – 60	15
61 – 70	9
71 – 80	6
81 – 90	4

7. The following histogram shows the frequency distribution of temperature (in °C) for each day of May 2010 in Sibi.
   (i)   What is the total number of classes?
   (ii) What is the width of the class?
   (iii) What is the total number of days?
   (iv) In which class the temperature is minimum?
   (v)   In which class the temperature is maximum?
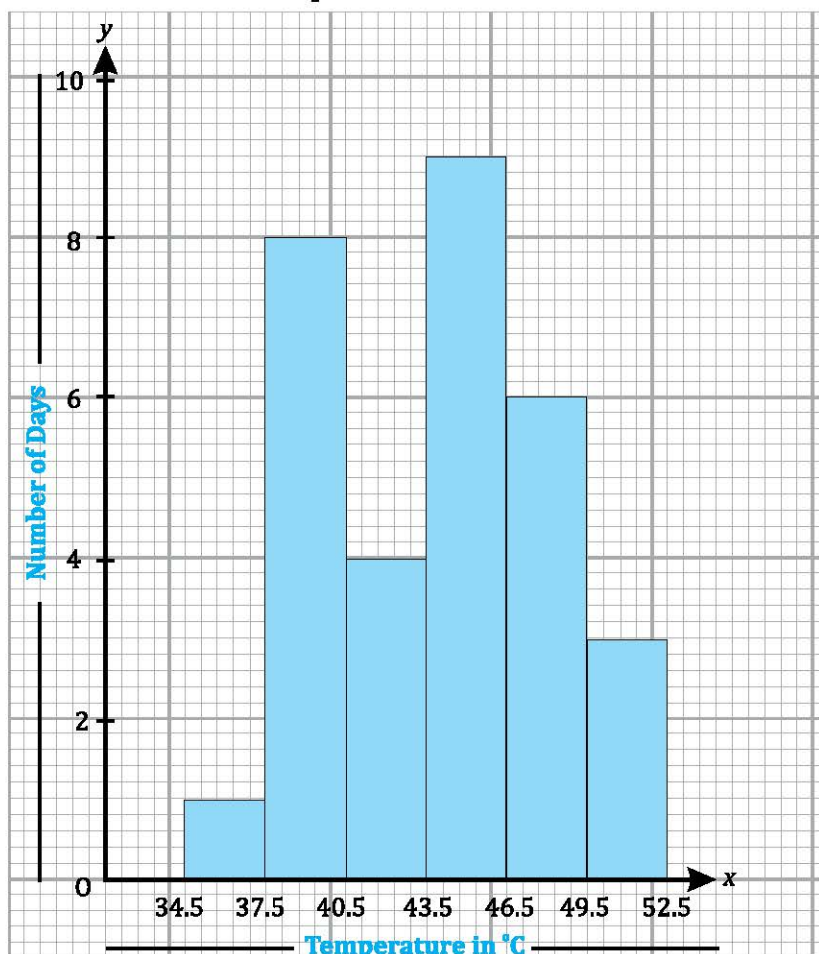
12.2 MEASURES OF CENTRAL TENDENCY

When the data is arranged into a frequency distribution, the observations contained in the data are easily understandable. We can find a single value which will represent all the values of a distribution in some definite way.

A central value that represents all the values of a data is called an average.

12.2.1 Description of measures of the central tendency

Averages lie in the central part of a data they are called measures of central tendency.

The most commonly used measures of central tendency are
(i)   Arithmetic Mean (Commonly Average)
(ii)   Median
(iii)   Mode

12.2.2   Calculation of mean, weighted means median and mode for ungrouped data

Arithmetic Mean

The sum of all the values of a set of data divided by the number of values is called arithmetic mean. It is denoted by $\bar{x}$.

$$\text{Arithmetic mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

If $x_1, x_2, x_3 \ldots x_n$ are n values in a set of data, then arithmetic mean can be calculated as.

$$\boxed{\bar{x} = \frac{\Sigma x_i}{n}}$$

Where "Σ" means sigma represents the sum of values and n is the number of values.

> Arithmetic mean is usually called average or simply mean.

Example 12.6 Find the mean of the values 8, 7, 12, 17, 9, 14, 10

Solution   We know that

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{8+7+12+17+9+14+10}{7}$$

$$\bar{x} = \frac{77}{7}$$

$$\bar{x} = 11$$

Example 12.7 The marks of 9 students out of 20 in a certain test are as follows:
9, 8, 16, 10, 7, 14, 15, 18, 11
Find mean marks of the students.

Solution
We know that

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\overline{x} = \frac{9+8+16+10+7+14+15+18+11}{9}$$

$$\overline{x} = \frac{108}{9}$$

$$\overline{x} = 12$$

Weighted Arithmetic Mean

Sometimes, different observations do not have the same importance. Some observations, for some reason, have greater importance. The relative importance called the weight (w) of the observations.

Let $x_1, x_2, x_3, \ldots x_n$ are the observations with corresponding weights $w_1, w_2, w_3, \ldots w_n$ then weighted arithmetic mean or weighted mean denoted by $\overline{x}_w$ can be calculated as.

$$\overline{x}_w = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n}{w_1 + w_2 + w_3 + \ldots + w_n}$$

$$\boxed{\overline{x}_w = \frac{\Sigma wx}{\Sigma w}}$$

Example 12.8 The marks obtained by Uzair in Mathematics, Urdu, English, History and Islamiat were 85, 80, 87, 68 and 65 with weights 9, 5, 3, 2 and 1 respectively. Find the weighted mean of the marks.

Solution

We know that

$$\overline{x}_w = \frac{\Sigma wx}{\Sigma w}$$

$$= \frac{(9)(85)+(5)(80)+(3)(87)+(2)(68)+(1)(65)}{9+5+3+2+1}$$

$$= \frac{765+400+261+136+65}{20}$$

$$= \frac{1627}{20}$$

$$= 81.35 \text{ marks}$$

Median

Median is the middle value of a set of observations arranged in ascending or descending order of magnitude. Half of the observations are above the median and half of the observations are below the median.

When finding the median, following steps should be kept in mind.

(i) Arrange the values in ascending or descending order.
(ii) If we have odd number of values in a set of data, the median is the middle mean.
(iii) If we have even number of values in a set of data, the median is the midpoint of two middle values.

Example 12.9 Find the median of the following observations.
(i) 15, 12, 17, 19, 14, 21, 16
(ii) 21, 24, 29, 22, 26, 35, 39, 32, 25, 36

Solution

(i) First of all, we arrange the values in ascending order.

12, 14, 15, (16), 17, 19, 21

Here $n = 7$

Since number of observations are odd i.e. 7

Therefore,

Median $= 4^{\text{th}}$ value

Hence, median = 16

(ii) First of all, we arrange the values in ascending order.

21, 22, 24, 25, 26, 29, 32, 35, 36, 39

$n = 10$

Since number of observations are even i.e. 10

∴ Median = Average of 5th and 6th observation

5th value = 26

6th value = 29

$$\text{Median} = \frac{26 + 29}{2}$$

$$= \frac{55}{2}$$

Median $= 27.5$

Mode

The most occurring value in a set of data is called mode. If each value occurs the same number of times, then there is no mode. If two or more values occur the same number of times but more frequently than any of the other values, then there is more than one mode.

Example 12.10 Find the mode of the following set of observations.
3, 6, 4, 5, 3, 4, 6, 7, 8, 4

3, 6, 4, 5, 3, 4, 6, 7, 8, 4
Since 4 is occurring 3 times in the data.
So, mode = 4

Example 12.11 Find the mode of the following data
23, 17, 19, 25, 17, 21, 19, 24, 17, 19

Solution
23, 17, 19, 25, 17, 21, 19, 24, 17, 19
17 and 19 are occurring 3 times each. So, Mode = 17, 19

12.2.3 Real life problems involving mean (average), weighted mean, median and mode

Example 12.12 The number of passengers on buses travelling on a certain route were recorded as shown below;
28, 27, 41, 46, 38, 40, 36, 40, 42, 35, 40, 37
Find the mean number of passengers.

Solution
28, 27, 41, 46, 38, 40, 36, 40, 42, 35, 40, 37
We know that

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{28+27+41+46+38+40+36+40+42+35+40+37}{12}$$

$$= \frac{450}{12}$$

$$= 37.5$$

Example 12.13 A student's final marks in Mathematics, Science, English and Islamiat are 80, 85, 90 and 75 with corresponding weights 5,4,3,3. Determine approximate average marks.

Solution   We know that

$$\bar{x}_w = \frac{\Sigma wx}{\Sigma w}$$

$$= \frac{(5)(80)+4(85)+3(90)+3(75)}{5+4+3+3}$$

$$= \frac{400+340+270+225}{15}$$

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$$= \frac{1235}{15}$$
$$= 82.33$$
$$\bar{x}_w = 82 \text{ marks} \quad \text{approx.}$$

Example 12.14 The median of a set of ten numbers is 10.5. Given that nine of the numbers are 9, 4, 3, 7, 15, 18, 14, 12 and 8, find the sixth number.

Solution

Let the sixth number $= x$

By arranging the numbers in ascending order

3, 4, 7, 8, 9, x, 12, 14, 15, 18

Since the number of values are even i.e. 10

Therefore,

Median = Average of 5th and 6th values

5th value = 9

6th value = x

$$\text{Median} = \frac{9+x}{2} \qquad \text{.......... (i)}$$
$$\text{But, Median} = 10.5 \qquad \text{.......... (ii)} \qquad \text{(given)}$$

By comparing eq. (i) and (ii)

$$\frac{9+x}{2} = 10.5$$
$$9 + x = 10.5 \times 2$$
$$9 + x = 21$$
$$x = 21 - 9$$
$$x = 12$$

sixth number = 12

Example 12.15 The scores of a basket ball team in a series of matches are 62, 64, 65, 64, 64, 51, 55, 57, 60, 54. Find mode.

Solution

64 appears 3 times among the ten scores.

So, Mode = 64

EXERCISE 12.2

1. Find the mean, median and mode of the following observations.
   (i) 14, 18, 12, 22, 26, 24, 15, 19, 22, 15
   (ii) 61, 66, 72, 64, 67, 75, 78, 65, 71, 68, 64, 74, 64

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(iii)   129, 135, 128, 129, 141, 142, 133, 137, 134, 145, 137, 141
(iv)   15, 16, 13, 6, 15, 17, 23, 16
(v)   47, 77, 87, 47, 67, 37, 47, 27, 57, 47, 44, 47
(vi)   2, 3, 4, 5, 6, 2, 3, 7, 8, 9, 4, 7
(vii)   3.5, 6.5, 4.5, 7.5, 5.5, 7.8

2.   The prices (in Rs.) of various Mathematics books are given below.
      18.50, 24.45, 34.65, 26.50, 44.05, 38.95, 56.40, 48.75, 29.30, 39.50
      Find the mean price of these books.

3.   The mean of six numbers is 45. Three of the numbers are 32, 35 and 42. The remaining three numbers each equals to x.
      (i)   What is the sum of six numbers?
      (ii)   Find the value of x.

4.   A student obtained 75 marks in English, 85 in Mathematics, 80 in History and 65 in Science. Find the weighted mean if weights of 4, 6, and 2 are allotted to the subjects respectively.

5.   Calculate the weighted mean from the following data.

Item	Expenditure	Weights
Food	270	7.5
Rent	50	1.5
Clothing	100	2.0
Fuel and Light	65	1.5
Other items	75	0.5

6.   The median of a set of ten numbers is 12.5 Given that eight of the numbers are 9, 11, 15, 10, 18, 14, 8 and 12, while the two missing numbers are x and y. Show that $x + y = 25$.

REVIEW EXERCISE 12

1. Choose the correct answer.
    (i)   The mean of the data 4, 5, 8, 7, 8, 4 is _____
            (a) 5   (b) 6   (c) 7   (d) 8
    (ii)   The mode of the data 2, 3, 5, 6, 4, 6, 7, 8, 6 is _____
            (a) 3   (b) 4   (c) 5   (d) 6
    (iii)   The upper class boundary of the class 41 – 50 is _____
            (a) 49.5   (b) 50.5   (c) 51   (d) 49

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(iv)   In a data, the largest value is 100, smallest value is 60 and size of class interval is 9 then there are _____ classes.
   (a) 5   (b) 6   (c) 7   (d) 8

(v)   If two or more values occur with the same frequency then each is called a _____
   (a) Mean   (b) Median   (c) Mode   (d) Histogram

(vi)   In the data, 4, 7, 12, 15, 22 the median is _____
   (a) 7   (b) 12   (c) 15   (d) 20

(vii) In the classes 41-45 and 46-50 the size of class interval is _____
   (a) 5   (b) 10   (c) 15   (d) 20

(viii) In the class 50.5-55.5, the lower class boundary is _____
   (a) 50   (b) 50.5   (c) 55   (d) 55.5

2. The following data shows the number of draws of different categories of prize bonds.
35, 55, 64, 70, 99, 89, 87, 65, 67, 38, 81, 80, 98, 51, 63, 66, 79, 85, 83, 70, 51, 99, 68, 95, 86, 53, 59, 50, 47, 55, 62, 60, 70, 78, 69, 86, 39, 71, 56, 75, 87, 64, 98, 62, 39
Construct a frequency distribution table by taking 10 as size of class interval. Also construct histogram of the grouped data.

3. Find the mean, median and mode of the following observations.

   (i)   48, 58, 42, 52, 57, 52, 59, 54, 59

   (ii)   205, 209, 196, 215, 225, 239, 205, 218, 186, 210

   (iii) 41.6, 48.4, 39.8, 42.3, 49.3, 39.4, 35.5, 38.5, 42.5, 41.6, 38.4

4. Arithmetic mean of 98 items is 60. Two items 60 and 80 were left out at the time of calculations. Find the mean of 100 items.

5. The following table indicates the increase in cost of living over July 2000, for a working class family as at 1st July 2004, and the weights assigned to various groups.

Groups	Percentage Increase	Weights
Food	30	8.5
Rent	65	3.5
Clothing	98.5	2.5
Fuel and lighting	75	1
Miscellaneous	80	4.5

Calculate weighted arithmetic mean of the data given above.

SUMMARY

➢ Frequency distribution is a table showing the number of items in each class.

➢ The number of times a particular items repeated in a given set of data is called frequency. It is denoted by f.

➢ A histogram is a vertical bar chart in which the rectangular bars are constructed at the boundaries of each class. Class boundaries are taken along x-axis and corresponding frequency along y-axis.

➢ Arithmetic mean is the sum of all the values in a set of data values divided by the number of values. i.e. Arithmetic mean $= \bar{x} = \dfrac{\text{Sum of values}}{\text{Number of values}}$

➢ If $x_1, x_2, x_3, \ldots x_n$ are the observations with corresponding weights $w_1, w_2, w_3, \ldots w_n$ then weighted arithmetic mean or weighted mean denoted by $\bar{x}_w$ can be calculated as $\bar{x}_w = \dfrac{\Sigma w_i x_i}{\Sigma w_i}$.

➢ Median is a middle value in a set of data after arranging the values in ascending or descending order. i.e. Median $= \left(\dfrac{n+1}{2}\right)$ th value.

➢ Mode is the most occurring value in a set of data.